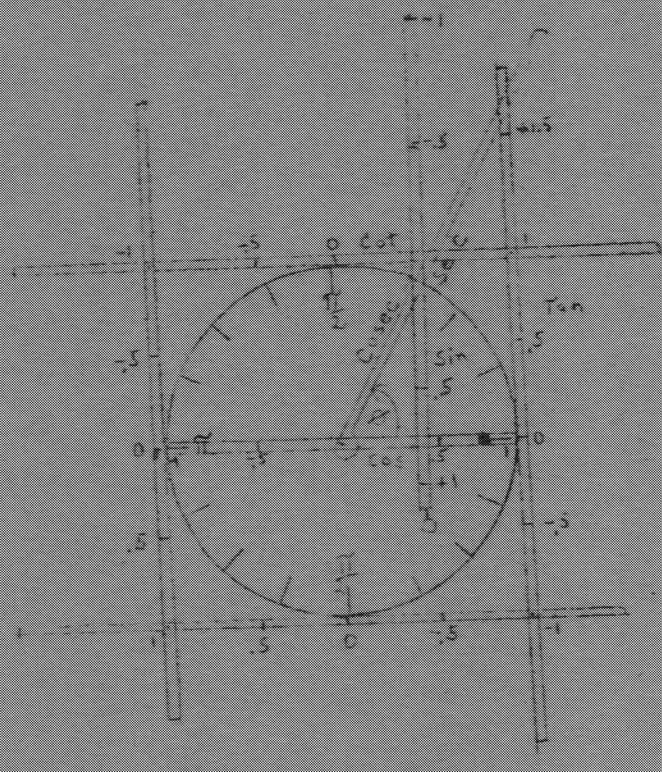
THE CONSTRUCTION AND USE

OF

MATHEMATICAL MODELS



The CONSTRUCTION and USE
of
MATHEMATICAL MODELS

by

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February, 1938.

I. Introduction

One of the principles often cited as basic in our system of elementary as well as secondary education is "learning by activity", or "learning by doing". In mathematics teaching in the primary grades, this principle is used in introducing the concept of number and familiarity with geometric shapes and figures. In the junior high school grades, it has not always been true that learning has continued by means of experimentation and exploration. Yet characteristic of the junior high school student is that he loves to experiment, he likes "to see it work", he wants to create and he enjoys exploring. One criticism that may be offerred of much of our mathematics teaching is that we have not permitted enough opportunities for exploration and discovery. Yet mathematics, more than any other subject, with perhaps the exception of science, is rich with such possibilities. A simple model or device, made by an otherwise uninterested student, may be the means whereby he is able to understand relationships, whose mastery leads to further investigations.

Among the models and devices which offer the best opportunities are those which can be made from simple and inexpensive materials, such as cardboard,wood, string, elastics, sticks, paper, etc. These can and should be made by every student. They are approximate in structure and of a temporary nature, but create interest and develop the imagination. Later, permanent models can be made from glass, metal, cellophane, wood, etc., which are neat, accurate, and strong in structure. Models of this type may be constructed as projects by various pupils and should form a part of a permanent mathematical collection for the school. They should lend themselves to rather severe handling and be used for exhibition and demonstration purposes.

Besides promoting a "spirit of discovery" and "creative thinking", the actual construction of a model requires a student to apply numerical and geometric principles previously studied, and thus gives an insight into applied mathematics. With this there can come a far better appreciation of the part played by mathematics in our modern industrial and scientific age. Students who cannot readily grasp abstract ideas and relationships, often sense them quickly when presented in concrete situations.

This pamphlet has been written to call to attention the value of models in mathematics instruction. The treatment is by no means complete or exhaustive. Many of the devices make the actual teaching of mathematics easier and the subject matter fascinating. They lead students to new ideas and discoveries, which in turn lead to still others. "It is fun to learn by doing."

II. Surveying Instruments

1. The Transit. Many junior high school textbooks contain chapters on indirect measurement--finding inaccessible distances by means of a home-made transit and a scale drawing. Directions for making a transit are included in many books.

If the unit on indirect measurement is properly developed, then the students usually find this work to be one of the most interesting in their mathematics course. Below are listed other simpler surveying devices which might well precede the work with the transit.

2. The hundreths of an inch scale. One means of arousing interest is to allow the studentsto make many simple instruments and devices of their own. The scale for measuring lines to hundredths of an inch is an example. If no ruler with an inch divided into tenth is available, an inch line can be readily divided into five equal parts by laying the line across six parallel lines (e. g., as found on eighth of an inch graph paper), and noting the four division points between the end points. Carefully bisecting the fifth of an inch, results in the tenth. The sides of a one inch square are now divided into tenths of an inch, and lines drawn parallel to the base, and obliquely to the sides, as shown on the scale included with this pamphlet.

3. The knotted rope. The Egyptians, before the day of the transit and compass, used to do their surveying with a knotted rope containing at least thirteen knots, equally spaced. The equally spaced knots aided in measuring distances, and if the rope was made to form a triangle with three, four and five spaces for sides, aright angle was obtained. Since most pieces of land were rectangular in shape, this earliest of surveying instruments proved quite satisfactory. Knotting a rope as noted above, is a good exercise in patience and accuracy for every student, and the importance of the right angle in surveying is emphasized.

4. The isosceles right triangle. Cut an isosceles right triangle out of stiff cardboard. Draw a perpendiculsr on each side of the cardboard, from the right angled vertex to the hypotenuse. Punch a hole on this line, near the vertex, and fasten a string with plumb bob attached, through it. Sighting the top of a telephone post, along the leg of this right triangle, keeping the plumb line in line with the perpendicular line drawn, makes it possible to obtain the height of thepost, within an error of 2 or 3 per cent. (Note theheight ofthe observer must be included in finding the height of the post.)

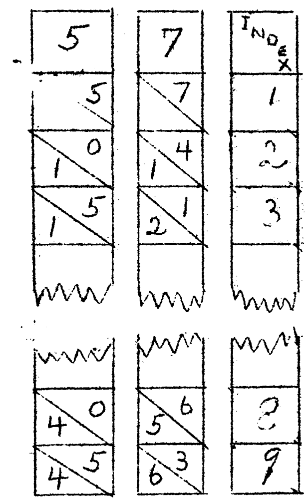
5. The rangefinder. Joseph's staff. These instruments are illustrated on Plate IV.

III. Calculating Instruments and Devices

1. The Abacus. This device is famousas one of the earliest of counting instruments. It is used to represent numbers in the decimal system and for performing the simpler processes of arithmetic. The addition, subtraction, and multiplication of positive integers is an interesting experience for junior high school students, particularly on the Chinese abacus, the suanpan. It is far more worthwhile for the studentsto make a simple abacus of their own, and use it, than for them to observe the teacher operating one. (Plate IV)

2. Napier's Rods or Bones. The invention of these rods by Napier, three hundred years ago, is probably the next step in the development of our modern calculating machine. These rods

or sticks can be made by using small strips of cardboard or wood, about the length and width of the "tongue Depressors" used by doctors and nurses. At least ten rods are needed. One of these is the index stick. It is divided into nine compartments, labeled from 1 to nine. The other nine sticks each contain a multiplication table of the number appearing at its top. The compartment is divided by a descending diagonal. The unit's digit of the tabular number appears in the top half of the compartment, and the ten's digit at the bottom. In Fig. 1, two ofthese sticks and an index rod are shown. To multiply 57 by 2, we obtain 114. 57 by 3 gives 171; note the adding of numbers 5 and 2 appearing in the same diagonal column. Again, 57 x 9 equals 513. To multiply 57 by 38, the products are obtained as above, but listed and added in the usual arithmetic form. Elementary and junior high school students delight in using their own Napier rods in drill lessons on multiplication.



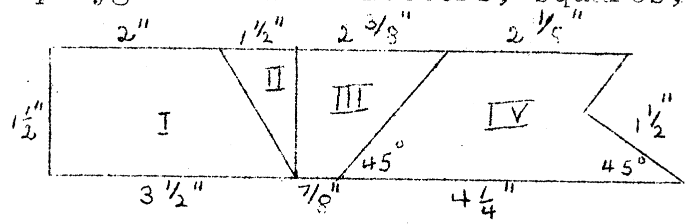
3. Numbergraph (Nomograph) for Addition and Subtraction. Materials required are a piece of cardboard, approximately 11" x 6", and a sheet of 11" x 8" graph paper, preferably 1/8" or 1/10" spaced. Cut the graph paper into one inch strips so that the heavy line isapproximately in the center of the strip. On the cardboard construct a 10" x 4" rectangle, accurately, and connect the midpoint of the 4" lines. Paste a strip of graph paper on each of the 10" lines, so that the heavy line of the strip lies directly on the line on the cardboard and the lowest cross lines of the strips correspond to each other along the 4" base line. Call the strips A, B, and C. The A and C scales are numbered alike, starting with 0 at the bottom and numbering EVERY OTHER division point with the consecutive numbers. On the B scale EVERY division is numbered consecutively beginning with 0 at the bottom. To add 3 and 5, place a foot ruler at the number 3 on A, and at 5 on C, and the answer 8 will be found at the intersection on B. To subtract, use the B scale for the minuend, the A scale for the subtrahend, and the result is found on C. A similar numbergraph based on positive and negative numbers proves to be an excellent device and aid in teaching ninth grade algebra.

Numbergraph for Multiplication and Division. This numbergraph is made in a similar manner to the one just described. Two types of semi-logarithmic paper are required,--one is the simple logarithmic scale of 10" length and the other has two logarithmic scales for the same length. Strips of paper are cut as before. Strips A and C require the single logarithmic scale and strip B the double. Care must again be taken that in glueing the strips to the cardboard, the lowest lines agree with the 4" base line. Number the A and C scales in the same manner as on the C scale of an ordinary slide rule, and the B scale like the B scale on the slide rule. To multiply 5 by 9, place the foot ruler at the point 5 on scale Aand at point 8 on C. The answer 40 (or 4) will be found on the second half of B.

4. The Slide Rule. A slide rule has recently appeared on the market retailing for twenty five cents. It appears to be satisfactory in introducing the usefulness of the slide rule to junior and senior high school students. However, a far greater understanding of the function and use of the slide rule will be gained if the student constructs one of his own, made out of wood or cardboard and costing a few cents. Strips of 10" single scale logarithmic paper may be used for the C, D and CI scales; double scale paper for the A and B scales, and triple scale 10" strips for the K scale.

IV. Geometric Figures

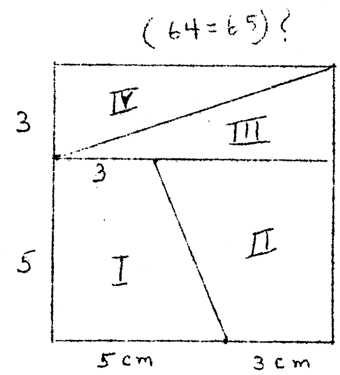
1. Cut-out puzzles. Among the many proofs of the famous Pythagorean Theorem, appear some which can be proved by fitting the parts of the squares on the legs into the square on the hypotenuse of the right triangle. Preliminary to this method of verifying or proving the theorem, students can be given practice in fitting together geometric polygons to form letters, squares, and entertaining figures. The letters A, E, F, H, M and T are among those commonly used. The letter T may be made from the four parts of the accompanying figure.

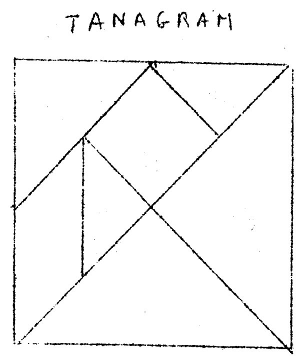


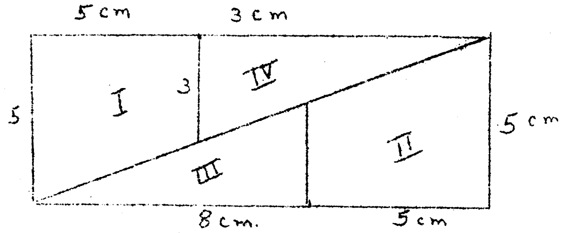
2. A Square equal to 20 Right Triangles. Twenty right triangles legs 2 cm. and 4 cm. can be made to fit a square. (This side of the square will be $8\sqrt{5}$ cm.)

3. 64 = 65? This famous problem belongs to this same group.

4. The Tanagram. This is a famous Chinese puzzle. To fit the seven pieces into a square is an interesting exercise. Other amusements are found in Dudeney, H. E., Amusements in Mathematics, p. 43-6.

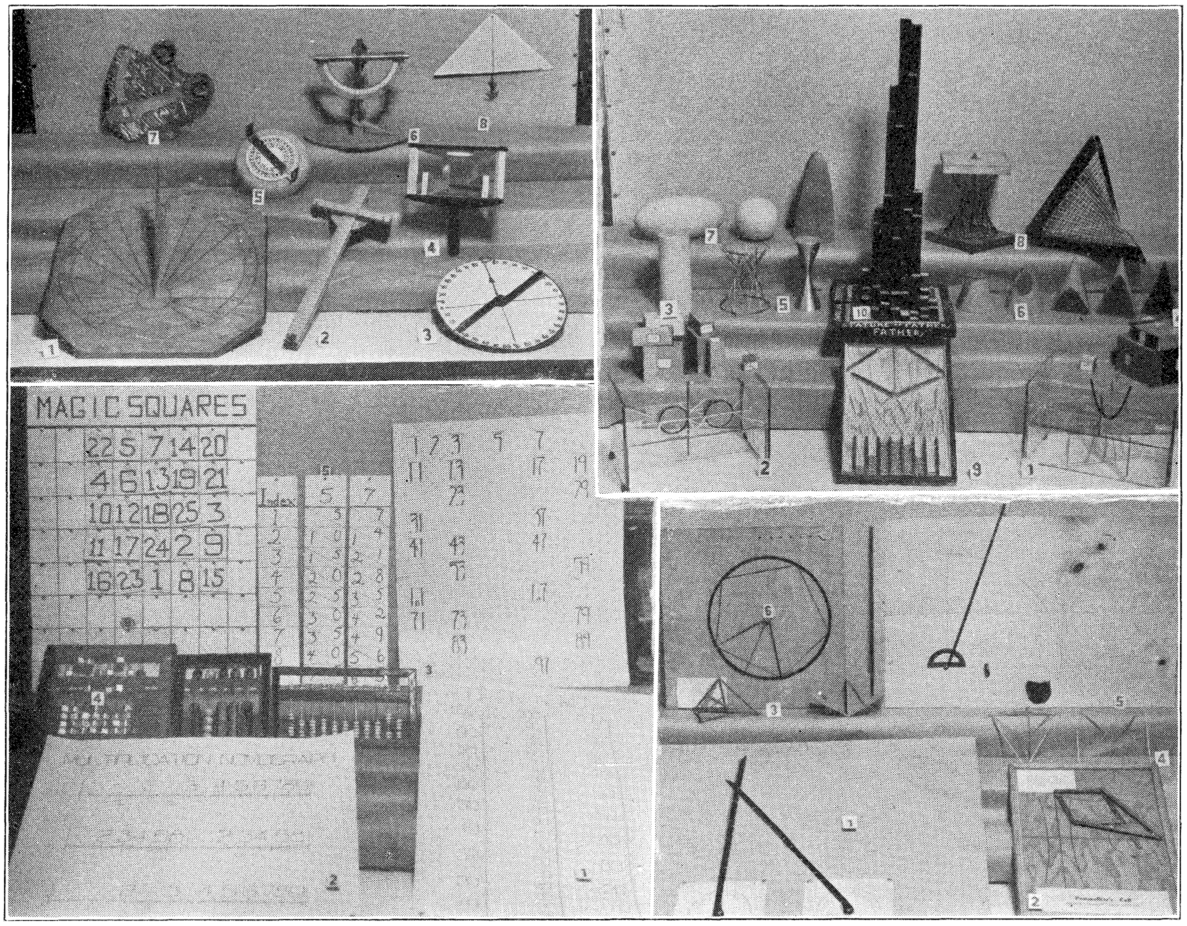






5. The Loculus of Archimedes. This consists of fourteen parts instead of seven. For figure, see Schaaf, W. L., Mathematics for Junior High School Teachers, p. 42.

6. Areas of Plane Figures. The parts of the accompanying figure can be rearranged to show that the area of a rectangle, parallelogram, trapezoid or triangle equals one half of the product of the sum of the bases and the altitude. (Figure on next p.)



CONSTRUCTION AND USE OF MATHEMATICAL MODELS

Plate V. Upper Left

1. Sun Dial
2. Joseph's Cross Staff
3. Astrolabe
4. Angle Mirror
5. Peloris
6. Theodolite
7. Sextant
8. Surveying Triangle

Plate IV. Lower Left

1. Addition Numbergraph
2. Multiplication Numbergraph
3. Prime Number Sieve
4. Abaci
5. Napier's Rods
6. Magic Square Board

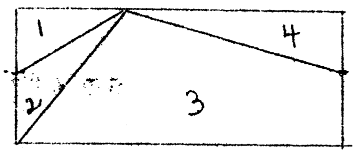
Plate III. Upper Right

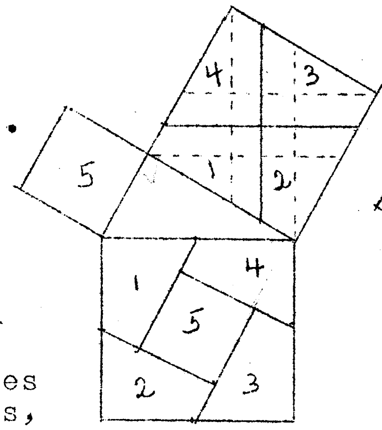
1. & 2. Complex Roots
3. & 4. Binomial Theorem
5. Generated Surfaces
6. Conic Sections
7. Solids of Revolution
8. Ruled Surfaces
9. Probability Board
10. Stereogram

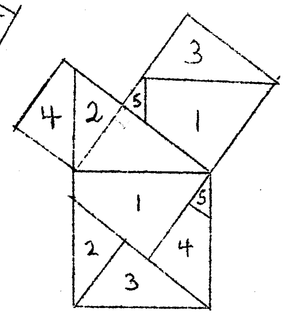
Plate II. Lower Right

1. Intuitive Geometry Board
2. Linkage
3. Solid Geometry Theorems
4. Frame Models
5. Trigonometric Functions
6. Theorem Board

7. The Pythagorean Theorem. The accompanying figures are self-explanatory. The parts of the squares on the legs, cut out of light cardboard, provide an interesting exercise in verifying this theorem.







V. Other Interesting figures.

1. Eratosthenes Sieve. A formula for expressing all prime numbers has never been found although it has been sought throughout the ages. A simple device used by Eratosthenes and known as Eratosthenes sieve for finding prime numbers, can be made into an interesting model. Construct a card A with 100 squares numbered from 1 to 100, preferably in rows of ten. A second card of the same size is made so that the small squares corresponding to the odd numbers, and to the number 2 on card A, are cut out ofthe cardboard. This card when placed over card A reveals all the odd numbers from 1 to 100 and the prime number 2. A third card has all the squares except those which are multiples of 3, cut out, i. e., 1,2,3,4,5, 7,8,10,11,13,etc. A fourth card has all spaces except multiples of 5 and a fifth card all spaces except multiplesof 7 removed. If these last four cards are placed on card A, only the prime numbers from 1 to 100 are visible. (Plate IV)

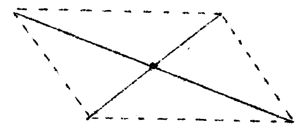
A variation of the above device is to have cards with every second square cut out, to show the numbers between 1 and 100 which are divisible by 2. Similar cards can be made for the other basic numbers of the common multiplication table.

2. Paper Folding. Many symmetrical figures can be obtained by folding paper. One of the simplest is to obtain either the sixteen or thirty-two points of the compass. Another is to obtain a regular pentagon, by tieing a simple knot on a strip of paper 1/2" or 1" wide. For other figures see Paper Folding by T. Sundara Row, Chicago, Open Court Publishing Company.

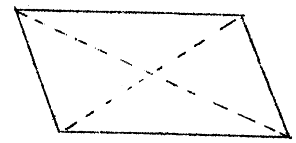
VI. Geometry

1. In geometry stress is laid on proving relationships in figures. These relationships are almost always given. It is of equal importance that a student discover relationships by experimentation, especially in the Junior High School, and later establish them logically in the demonstrative geometry. To this end models with movable parts, to give general relationships, are invaluable. Fixed models are useful in discovering particular relationships. The following models are merely indicative and many others as simple or more complex can be devised by enthusisatic teachers and students.

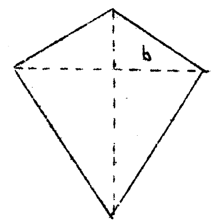
2. Parallelogram: Join two sticks of unequal length by a single bolt at their mid-points. Stretch an elastic tape around the edges. As the sticks are moved in any position the elastic band always forms a parallelogram.



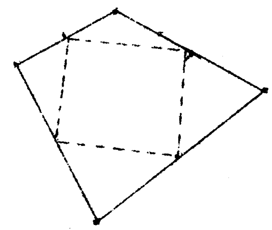
3. Diagonals: Join two unequal pairs of equal sticks, alternately, by single bolts. Fasten elastic diagonals. For any position of the sticks the diagonals bisect each other. The locus of the diagonals' intersection, for a fixed base is easily seen from the model.



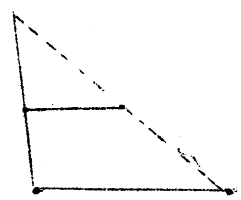
4. The Kite: Join the unequal pairs of equal sticks, successively, and fasten elastic diagonals. The elastics are always perpendicular for any position of the sticks. The locus of the diagonals' intersection, when the position of one diagonal (b) is fixed illustrates the perpendicular bisector of a given line segment.



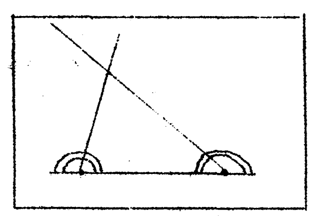
5. Quadrilateral: Join the ends of four unequal sticks with interlocked eye hooks. Insert an eye hook at each mid-point and through them pass, in order, an elastic loop. The loop forms a parallelogram no matter in what position, plane or in space, the four sticks are held.



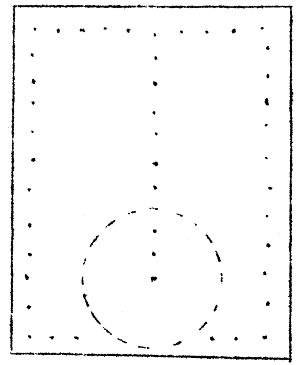
6. Triangle: On a drawing board, thumb tack the ends of two unequal elastic cords, separately, ten units apart. Connect the mid-points of the elastics by a non-elastic cord or stick 5 units in length. Joining the loose ends of the elastics to a single point and moving this point to any position in the plane of the board so that a taut figure is formed, the 5 inch cord is always parallel to the base. What happens in this same figure if the thumb tacks are 12" apart?



7. Intuitive Geometry Board: Divide a large board 30" x 20" into unit squares. On a lower horizontal line attach or paste two protractors (celluloid or transparent if possible) with centers 10 units apart. To each center attach metal strips, divided into units, and numbered, zero in each case at the protractor center. A surprisingly large number of theorems relating to triangles and parallel lines can be discovered on this board. A sliding protractor on one arm can be an added feature. (See Plate II, no. 1)



8. Theorem Board: At the bottom center of a board 20" x 20" paint a black circle, radius 5". Around the board, around the circle, at the center, and along a vertical diameter drill small holes, to fit golf tees. Drill as many holes as possible,

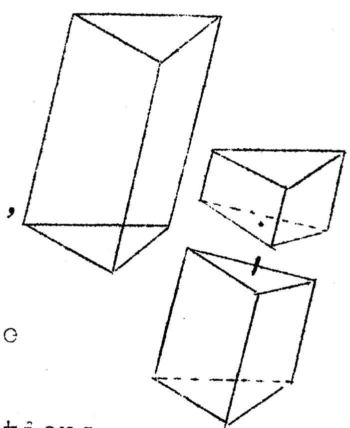


symmetrically placed. With red and black colored elastic, and golf tees with lettered heads, any plane geometry theorem relating to a single figure can be represented. (See Plate II, no. 6)

Many other models on angle measurement, similarity, circles, the Pythagorean theorem, etc., are described in various texts and form excellent teaching devices.

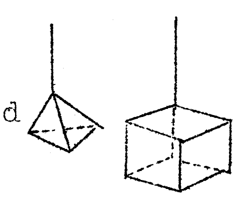
9. Solid Geometry Theorem Models: In the beginning of the study of solid geometry, models of the theorems are especially important. They serve the same purpose as a blackboard drawing of a figure in plane geometry, which is essentially a model for the theorem. Temporary models can easily be made from such materials as cardboard, pencils, knitting needles, thin sticks, cork, string, and paste. With these materials every student can make his own model of the theorem under discussion. Permanent models can be made from wood, glass, heavy celluloid or wire mesh. (See Plate II, no. 3)

10. Geometric Solids: Models of prisms, pyramids, cones, cylinders and spheres can be turned out of metal or wood on a lathe in the school shop. Sections of these solids (right, oblique, circular, elliptical, etc.) can be sawed through, the cut filled with shellac and the parts connected by counter-sunk pins. In Junior High School these models can be measured, to determine area and volume relations as well as to establish the concept of approximate computation and degree of accuracy. (See Plate I, no. 8)



The models can be made out of slender rods, wooden or wire, with the ends sealed with wax, glue or solder. Such models aid in discovering and visualizing diagonal and interior relationships. (See Plate II, no. 4) The solids can be made out of sheeted tin or celluloid, with one part open, and in this manner with the use of sand or liquid, be used to establish volume relations, as well as to afford an interior view of the solids. Celluloid models of intersecting solids, such as two cylinders, are valuable, as such intersections are exceedingly difficult to visualize. (See Plate I, no. 5)

11. Soap bubble figures: The sphere and intersecting spheres are easy to make from a hollow tube and soap water. A regular tetrahedron, constructed with six thin metal rods as edges and attached to a slender rod at one vertex, when dipped in a soap solution will produce a bubble in the form of an intraverted tetrahedron. (See Plate I, no. 2) A cube similarly constructed will produce an even more striking geometric space surface.



12. Regular Polyhedra: Many books give templets for constructing the regular polyhedra, (Plate I, no. 1). That other interesting and important polyhedra exist and can be made from a single templet is seldom noted. All crystals are polyhedra containing symmetry and models of them are given in many books on crystallography, (See Plate I, no. 6). The simplest one of these is a rhombododecahedron or extraverted cube, consisting of twelve faces all congruent rhombi. To construct it connect each vertex of a cube to its center and fold the six square pyramids thus formed around the outside of the cube. A templet for this solid is given on the last page. All the regular polyhedra can be treated the same way, thus creating an entire new class of polyhedra, (See Plate I, no. 3). If the pyramids are intraverted we get an interesting new set of geometric figures, which have faces, edges and vertices, yet inclose no space. (See Plate I, no. 2)

13. Stellated Polyhedra: If the faces of the regular octrahedron, dodecahedron and icosahedron are extended till they meet, a new set of pointed or stellated polyhedra are formed. (See Plate I, no. 4) A templet for the stellated icosahedron is given on the last page. In the case of the icosahedron the pyramids can be intra- or extra-verted. If, in turn, these stellated polyhedra have their faces produced until they meet, a new set of polyhedra (stellated of the second order) are formed. The construction of the templets and the figures is an excellent project for a solid geometry class. These figures are described in detail in Max Bruckner's Vielecke und Vielflache, published by G. B. Teubner, Leipzig. Some are described in Craftmanship in the Teaching of Elementary Mathematics by F. W. Westaway, page 584.

14. Penetrating Solids: Many crystals are formed by the intersection of two geometric solids. An interesting set of solids are those formed by taking the regular solids formed according to the same scale, and letting one intersect the other along symmetric axes. All the above solids when carefully constructed make beautiful ornaments for display purposes.

15. Euler's Theorem: For all these solids, the important theorem $E + 2 = V + F$, known as Euler's polyhedron theorem, can be easily verified. A good model to aid in the proof is to place part of a wire polyhedron on a circular base under a hemispherical glass dome (a fish bowl cut by a glazier will suffice). From any point on the circular base inside the polyhedron let wires be taken through each vertex to the globe surface. Through these points of contact, paint arcs of great circles to correspond to the edges of the polyhedron. (See Plate I, no. 9). The study of the spherical surface then suggests a method of proof. Teachers can devise models for use in the proof by synthetizing the solid, or by projection.

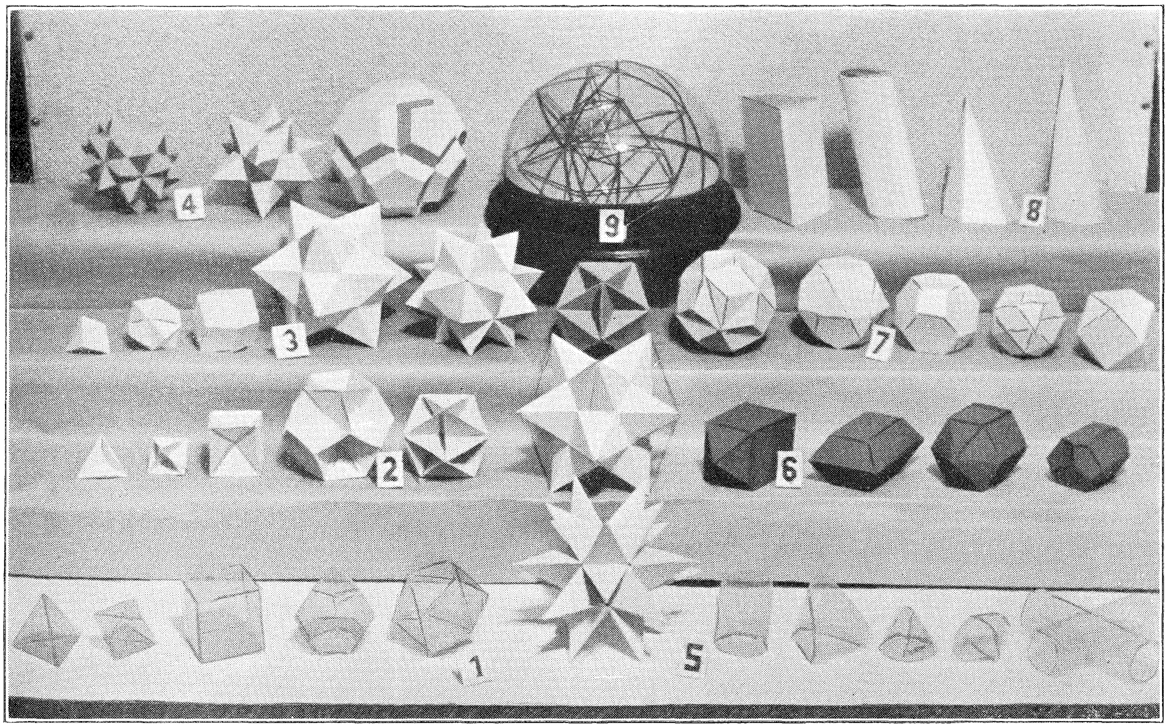


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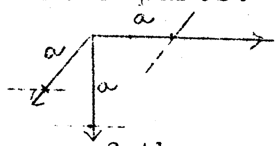
Plate 1

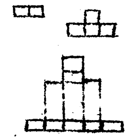
1. Regular Polyhedrons
2. Intraverted Polyhedrons
3. Extraverted Polyhedrons
4. Stellated Polyhedrons
5. Intersecting Solids
6. Crystals
7. Archimedean Solids
8. Generated Solids
9. Euler's Theorem

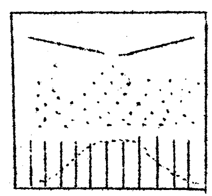
VII. Algebra and Analysis

1. Binomial Theorem: $(a + b)^2$ and $(a - b)^2$ are represented geometrically in many texts by a aquare, cut into four parts. Similarly $(a + b)^3$ can be constructed from a cubical box by making three complete cuts, each at the same distance from one given vertex and perpendicular to the edge. The eight solid sections thus formed correspond to the terms of the expansion of $(a + b)^3$. By properly relabeling the parts so that a + b equals x and b = y, and hence a = x - y, the same model will illustrate $(x - y)^3$. (See Plate III, no. 3 and 4.) The students can also devise models for (a + b + c) squared or cubed, and the whole process leads later ot Symmetric Functions.

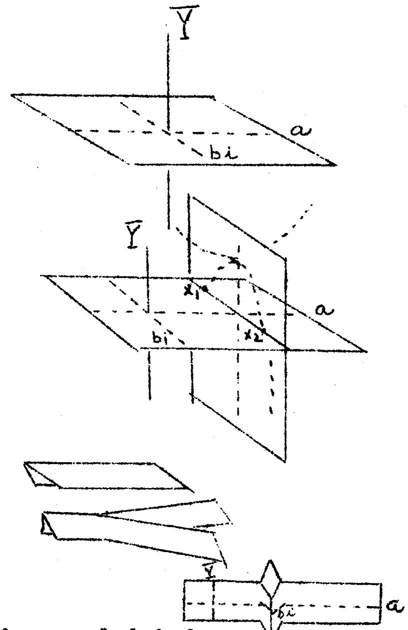


2. Pascal's triangle: As an extension of the binomial theorem and for the numbrical relationship obtainable from it, this triangle is interesting in itself. To extend it to the normal curve, it is of value to build each row separately, out of equal blocks as 1-2-1, 1-3-3-1, etc. What happens when n becomes large can be illustrated by Galton's Quincunx. (See Plate III, no. 9) The simplest type is that of a penny machine in which round pegs are driven into a board in ten-pin or equilateral triangular arrangement with sufficient space for a penny to fall through. Have at least 20 spaces at the bottom. Dropping pennies through the center slot at the top and with the board sufficiently inclined to let the penny slide, the columns will fill so that the tops of the pennies form a nearly normal curve.



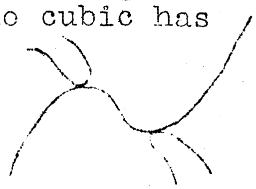


3. Complex Roots of a Real Equation: The real roots of a quadratic equation can be easily represented on a graph of the function. In solving quadratic equations, the student frequently gets complex roots which the ordinary graph of the function does not show. After complex numbers have been taught, the complex values of x that give real values to y in $y = Ax^2 + Bx + C$, can be graphed. (See Plate III, no. 1 and 2.) For this purpose we use a real axis Y and a complex plane X = a + bi, perpendicular to the Y axis, the origin being their intersection. Solving for x in terms of y, then substituting any real values for y, we get the corresponding values for x, some real , others complex. Setting the discriminant equal to zero, one obtains the least or greates value of y that gives a real x. At this corresponding value of x, we construct a plane MN perpendicular to the 'a' axis. This plane contains all complex values of x, corresponding to real values of y. The complete graph has then two parts, the usual parabola in the real Y → 'a' plane and another parabola in the MN plane joined at their vertices. The intersection of the complete graph by the X or a + bi plane gives the roots of the quadratic, both real, or both complex. The model can easily be made from three glass planes with the parabolas painted, preferably in contrasting colors.

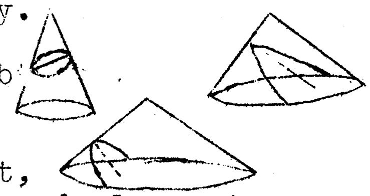


Every student in the class can make his own model from ordinary graph paper. Bend a piece of graph paper lengthwise. Select the origin and determine the bend point value of x and slit the paper along the crease to this bend point value. Crease each slit part into three equal parts, fold the two interior sections of each part together placing the two third sections together and in the same plane as the unslit part. The two middle sections comprise the complex plane. Label the real and imaginary axis and graph the curve on all visible sides. Looking towards the front only the real part of the curve is seen; from the side only thecomplex part. In a glass model both parts are readily visible.

The same structure can be carried to cubic and higher equations. The model is similar except that the cubic has its complex values in two hyperbolic concave surfaces instead of a plane. (See Plate III no. 1). On such a glass model, all the n roots, real or complex, are represented.

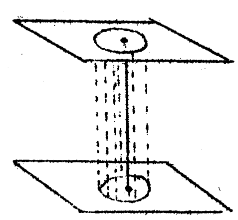


4. Conic Sections: All the conic sections can easily be cut from a cone (see Plate III, no. 6). A double napped cone is best and can be joined at the vertices by a counter sunk pin. The conic sections can also be made from paper by cutting from circles of radius, r, three sectors whose arc lengths are less than, equal to, and greater than $\pi r \sqrt{2}$ respectively. Pasting the two edges together there arises an acute angled, right angled and obtuse angled cone respectively. Cutting through the cones perpendicular to a slant height, the cut will result in an ellipse, parabola and hyperbola, respectively.



5. Ellipse: Around two round headed thumb tacks, on a drawing board, place a looped, non-elastic cord. Placing a pencil inside the loop and moving the point so as to keep the cord taut, an ellipse is traced. As the thumb tacks are placed closer together, the ellipse approaches a circle.

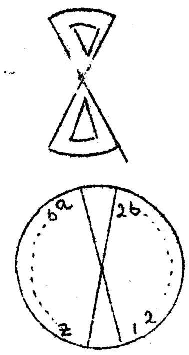
6. Ruled Surface: Between a double napped cone and a cylinder there are intervening figures. (See Plate III, no. 5 and 8.) The transitions can be shown by the following model. Separate two blocks, on which have been drawn circles of diameter 5 inches, by an eight inch bar, fixed to the center of one circle and clamped at the other center. Perforate the circumferences of the circles with small holes about one half inch apart. Lace with elastic bands so that each band is vertical, thus forming a cylindrical surface. As the loose block is twisted, the cylinder is changed to single sheeted hyperboloids and finally to a double napped cone.



7. Trigonometry: To show the variation in the trigomometric functions in all four quadrants, the following model is most convincing. On a board 30" x 30", mark the center and paste a 10 inch radius, complete 360° protractor. Let 4 measuring tapes 30" long, marked from -15 to +15 be pasted parallel to the 0°-180° and 90°- 270° axes and tangent to the circle. Attach to the center a rotatable metal strip marked from 0 to 20 inches. On it, at the 10 inch division, attach a freely hanging measuring stick marked from 0 to 10 inches. As the arm rotates the variation in all the functions can be easily followed. (See Plate II, no. 5)

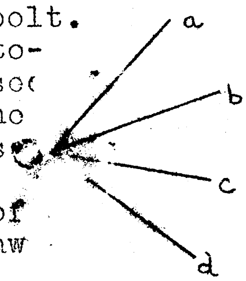
VIII. Recreation and History

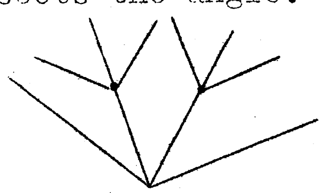
1. Numerology: Gematria or assigning numerical value to words is one of the most ancient practices. A simple board to illustrate this is made from at least 5 concentric circles. Divide each circle into two sectors, each slightly less than a semi-circle, by two intersecting diameters. Divide each of these sectors into 26 corresponding equal parts. In the one sector write the alphabet, and in the other in reverse order the numerals 1 to 26. Place a double sector shield, slotted, between the sectors containing the numerals and letters. Arranging the circles so that the letters in the one slot spell any 5 (or less) letter word, the numerical values appear on the opposite slot of the shield.

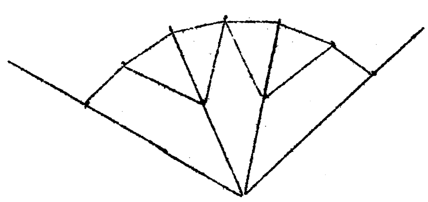


2. Magic Square Board: Divide a board into 10 by 10 equal squares with a hook on each square. Cut numbers from 1 to 100 and place on separate cards whose size is the same as that of the square on the board. Perforate to fit hook and to cover the square. Then a magic square of any order 3 to 10 can be represented. (See Plate LV, no. 6)

3. Trisecting the Angle: There are many mechanical devices for solving or approximating the three famous problems of antiquity. A simple model for trisecting an angle is to take four sticks, 8 inches in length, slotted through at the middle longitudwise, and fasten all four at one end to one bolt. Take 4 sticks of 7 inch length. Bolt two of them together at the end and fasten bolt through slot of second arm.(b). Do likewise with the other two and the third arm.(c). Connect the outer loose ends successively by six equal sticks about 4 or 5 inches in length. If the center (0) is placed at the vertex of any angle, and the outer slots along the sides, drawing a pencil point along the two inner slots trisects the angle.







IX. Other Models

There are many other models which are valuable but lack of space permits no explanation of them. Among such models are map projection, stereographic projection, the sextant, (Plate V, no. 7), sun dial, (Plate V, no. 1), stereograms, (Plate III, no. 10), linkages, (Plate II, no. 2), magic cubes, duplication of the cube, and inscrebed regular polygons. Also models for the non-Euclidian geometry parallel postulate, the tesseract and fourth dimensional geometry, projective geometry, spherical trigonometry, and astronomy. The field is rich and fruitful and every teacher of Mathematics can find nuch aid in it. This pamphlet serves merely as a guide and an introduction to what should prove to be a more fascinating method of mathematical instruction.

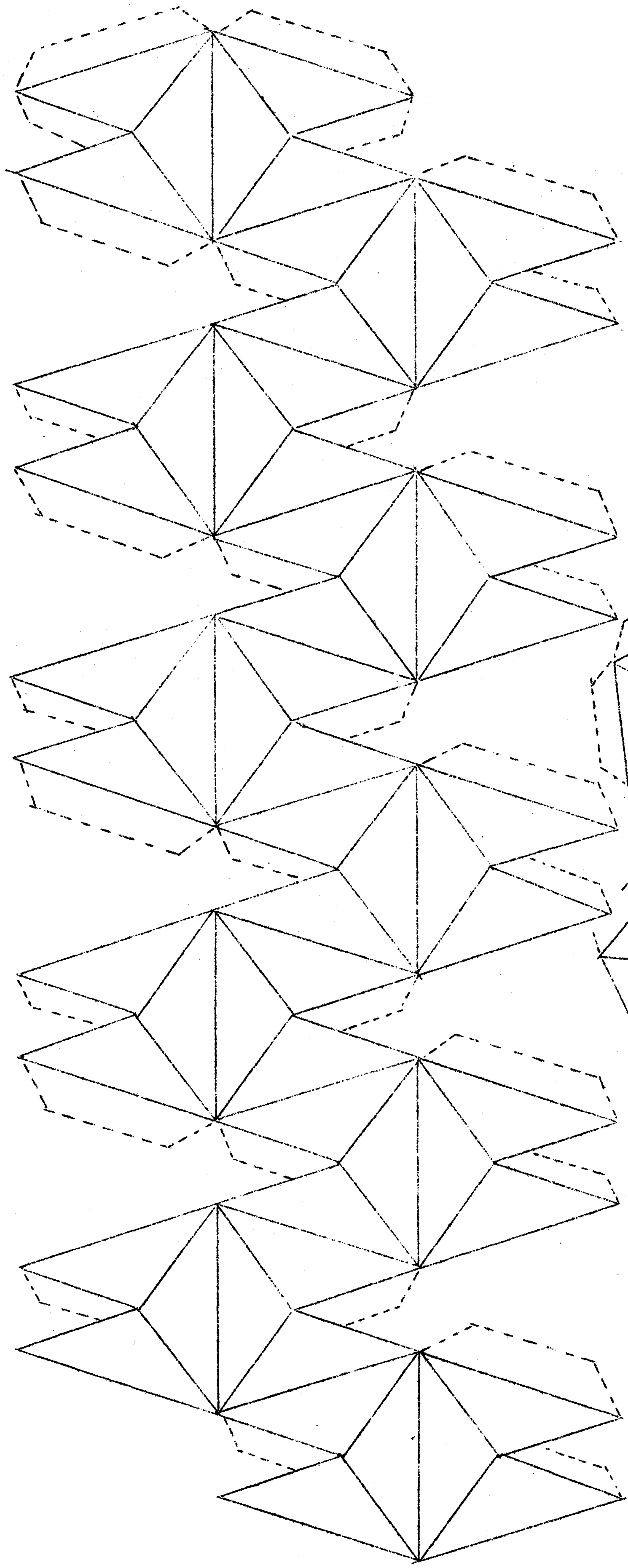
X. Suggestions on Materials

1. In making stiff paper models use index card and not cardboard. Index card comes in large sheets of various colors, is light, strong, flexible and easy to handle. Library paste or other quick drying paste should be used. All edges should be cut by a sharp knife or razor blade, sliding along a straight edge. Do not use a scissors.

2. In using celluloid for plane surfaces, use the heaviest sheeted material. For curved surfaces use the light rolled celluloid. The best sealing material is Dupont's glue. Celluloid models are clear, transparent, and lasting but must be handled with care.

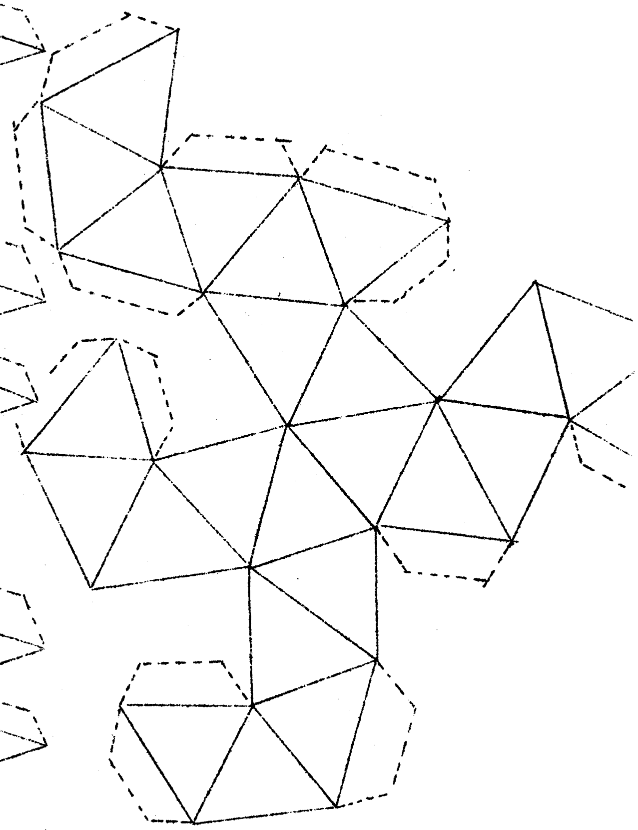
3. Ordinary plate glass is the best kind for glass models. Measurements should be taken in advance and all glass parts cut to size by a glazier. Glass cement or furniture glue is the best sealing substance. While such models are heavy, easily broken, clumsy to handle, they make excellent stationary models because of their transparency.

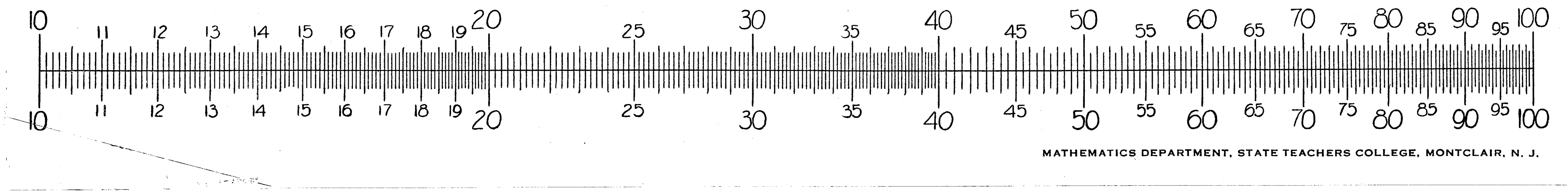
4. Wood and metal are the easiest materials to handle and can be turned on a lathe. Models made from them are permanent and good for demonstration purposes.



Stellated Icosahedron,
Intra- or Extraverted.
Fold longer solid lines outward, shorter solid lines inward. Paste short lips under adjoining face. Fold either way, pasting longer lips to adjacent faces.

Rhombododecahedron,
Intra or Extraverted
Cube





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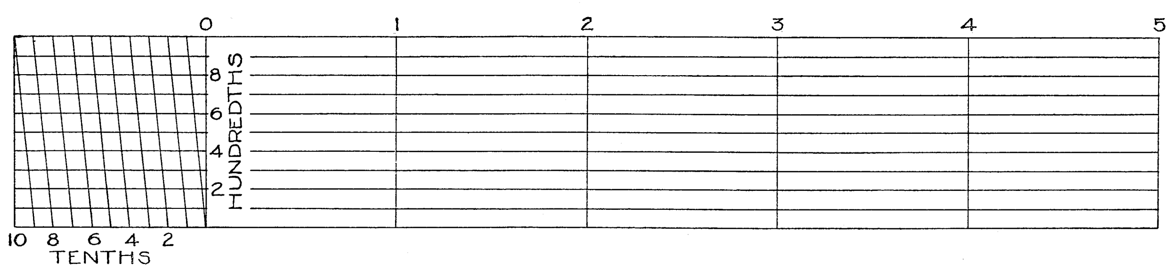

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